Instructions: Complete each of the following exercises for practice.

1. Find the limit (if it exists), or show it does not exist.

   (a) $\lim_{(x,y)\to(3,2)} (x^2y^3 - 4y^2)$

   (b) $\lim_{(x,y)\to(2,-1)} \dfrac{x^2y + xy^2}{x^2 - y^2}$

   (c) $\lim_{(x,y)\to(0,0)} \dfrac{x^4 - 4y^2}{x^2 + 2y^2}$

   (d) $\lim_{(x,y)\to(0,0)} \dfrac{5y^4 \cos^2(x)}{x^4 + y^4}$

   (e) $\lim_{(x,y)\to(0,0)} \dfrac{xy}{\sqrt{x^2 + y^2}}$

   (f) $\lim_{(x,y)\to(0,0)} \dfrac{x^3 - y^3}{x^2 + xy + y^2}$

   (g) $\lim_{(x,y)\to(0,0)} \dfrac{y^2 \sin^2(x)}{x^4 + y^4}$

   (h) $\lim_{(x,y,z)\to(\pi,0,\frac{1}{3})} e^{y^2} \tan(xz)$

   (i) $\lim_{(x,y,z)\to(0,0,0)} \dfrac{xy + yz}{x^2 + y^2 + z^2}$

2. Compute $h(x, y) = g(f(x, y))$ and find the set of all points at which h is continuous.

   (a) $g(t) = t^2 + \sqrt{t}, \quad f(x, y) = 2x + 3y - 6$

   (b) $g(t) = t + \ln(t), \quad f(x, y) = \dfrac{1 - xy}{1 + x^2y^2}$

3. Determine the set of points at which the function is continuous.

   (a) $f(x, y) = \dfrac{1 + x^2 + y^2}{1 - x^2 - y^2}$

   (b) $g(x, y) = \dfrac{e^x + e^y}{e^{xy} - 1}$

   (c) $h(x, y) = \ln(1+x-y)$ $k(x, y) = \arcsin(x^2 + y^2 + z^2)$

4. Use polar coordinates to compute the limit (**Hint**: As $(x, y) \to (0, 0)$ you can always have $r \to 0^+$...).

   (a) $\lim_{(x,y)\to(0,0)} \dfrac{x^3 + y^3}{x^2 + y^2}$

   (b) $\lim_{(x,y)\to(0,0)} (x^2 + y^2) \ln(x^2 + y^2)$

   (c) $\lim_{(x,y)\to(0,0)} \dfrac{e^{-x^2-y^2} - 1}{x^2 + y^2}$

   (d) $\lim_{(x,y)\to(0,0)} \dfrac{\sin(x^2 + y^2)}{x^2 + y^2}$

5. Prove that the function $f(\mathbf{x}) = \mathbf{v} \cdot \mathbf{x}$ is continuous on $\mathbb{R}^n$ for all $\mathbf{v} \in \mathbb{R}^n$.

6. Prove that the function $f(\mathbf{x}) = |\mathbf{x}|$ is continuous on $\mathbb{R}^n$. (**Hint**: What type of function is $|\mathbf{x}|^2$?)